Journal of Applied Mechanics and Technical Physics, Vol. 41, No. 4, 2000

EXPERIMENTAL STUDY OF THE STRENGTH OF COMPOSITE PLATES IN BENDING

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UDC 239.374

The strength of composite plates in bending and the effect of the angular parameters at the tips of the joint of the constituents on the strength are studied. An approximate experimental finite-stress curve ($\lambda = 1$) that separates the low-stress zone from the stress-concentration zone near the angular point of the joint is obtained in the plane of the angles (α, β) by analyzing the photoelastic patterns (isochromatic curves), fracture lines, and rupture moments. A comparison with the theoretical curve obtained from the solution of the corresponding problem is performed. An experimental relation between the strength of the plate and the angular parameters of the constituents is obtained for the stress-concentration and low-stress states near the joint tips.

Introduction. Theoretical studies of the strength of the interface edge in a composite body within the framework of the low-stress problem enable one to establish a relation between the characteristics of the material in tension and the opening angles of the constituent wedges [1]. The limiting curves that separate the low-stress zones from the stress-concentration zones in the coordinate plane of the angles depend only on the physical and geometrical parameters of the materials. However, the strength of the real adhesive joints of composite bodies depends on the methods of fabrication, the thickness of an adhesive layer, and its strength properties. To verify theoretical findings, it is necessary to perform experimental studies with allowance for real conditions under which the layered body is deformed.

1. Formulation of the Problem. Theoretical low-stress investigations in composite plates in bending allow one to study the low-stress zone in the neighborhood of the corner edge of two joined materials depending on the mechanical and geometrical parameters [2, 3]. This makes it possible to avoid the stress concentration by varying the geometrical configurations and determine the strength criteria that depend on the geometrical and physical parameters.

The aim of the present study is to show experimentally the existence and location of the low-stress and stress-concentration zones at the angular points of composite slabs (plates) in the plane of geometrical variables of the constituents. Another aim is to calculate the intensity of the critical moments which cause failure at the angular points. If the stress concentration occurs at these points, failure can occur at small loads.

2. Equation of the Limiting Curve. Let a wedge-shaped plate of thickness h made of exponentially strengthening epoxide and Duralumin be subjected to transverse bending (α and β are the wedge angles of the epoxide and Duralumin constituents, respectively) (Fig. 1). A cylindrical coordinate system is introduced on the neutral surface of the plate. Expanding the displacements into Taylor series in powers of z, retaining the first two terms, using relations of the theory of elastoplastic deformations and the equations of equilibrium, and taking shear strains into account, we obtain the expressions for the moments and transverse forces

Institute of Mechanics, Armenia National Academy of Sciences, Erevan 375019. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 4, pp. 211–215, July–August, 2000. Original article submitted May 27, 1998; revision submitted August 23, 1999.

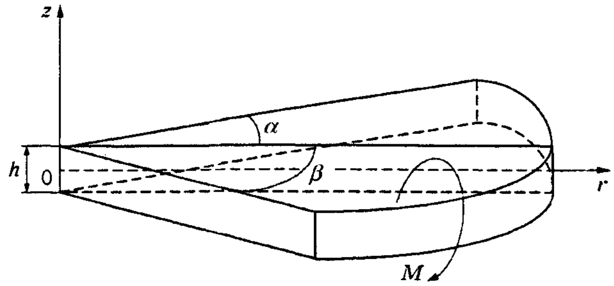

Fig. 1

$$M_{ij}^{(k)}(r,\theta) = r^{(\lambda-1)m} F_{ij}^{(k)}(\theta,\lambda), \quad Q_i^{(k)}(r,\theta) = r^{(\lambda-1)m+1} \Phi_i^{(k)}(\theta,\lambda),$$

where the superscripts $k = 1$ and 2 correspond to epoxide and Duralumin, respectively, the subscripts i and j correspond to r and θ, respectively, λ is the eigenvalue of the problem, and the functions $F_{ij}^{(k)}(\theta,\lambda)$ and $\Phi_i^{(k)}(\theta,\lambda)$ are expressed in terms of eigenfunctions by nonlinear relations. In the neighborhood of the angular point, the main stress components have the form

$$\sigma_{ij}^{(k)}(r,\theta) = \frac{M_{ij}^{(k)}(r,\theta)z}{J}. \tag{1}$$

We consider the boundary conditions

$$M_\theta^{(k)}(r,\theta) = 0, \quad M_{r\theta}^{(k)}(r,\theta) = 0, \quad Q_\theta^{(k)}(r,\theta) = 0 \tag{2}$$

for $\theta = \alpha$, β and the continuity conditions

$$M_\theta^{(1)}(r,0) = M_\theta^{(2)}(r,0), \quad M_{r\theta}^{(1)}(r,0) = M_{r\theta}^{(2)}(r,0), \quad Q_\theta^{(1)}(r,0) = Q_\theta^{(2)}(r,0) \tag{3}$$

at the interface $\theta = 0$.

As a result, we obtain an eigenproblem for a 12-order system of nonlinear ordinary differential equations subject to the homogeneous boundary and contact conditions. A numerical solution of the above-formulated problem yields the eigenvalue λ which is a function of the physical and geometrical parameters of the composite plate: $\lambda = \lambda(\alpha,\beta,m,\gamma)$, where γ is the ratio of the bulk moduli of the materials and $m = 1/n$ (n is the degree of strengthening, which is the same for both materials). The relation between the above-mentioned parameters can be determined by specifying the value of λ and solving the inverse problem. Setting $\lambda = 1$, we determine the limiting curve in the coordinate plane (α,β). This curve separates the low-stress zone from the stress (moment)-concentration zone. In the limiting case of linear-elastic materials ($m = 1$), we obtain the following equation of the limiting curve:

$$\frac{\sin 2\alpha}{1 + 3\cos 2\alpha} + \gamma\frac{\sin 2\beta}{1 + 3\cos 2\beta} = 0. \tag{4}$$

3. Experiment. In the experiment, the stresses at the angular points of the wedge-shaped plates whose opening angles satisfied the condition $\alpha + \beta = \pi$ (Fig. 2) were studied.

Strength was studied experimentally with the use of specimens of composite plates with Duralumin ($E_D = 7.1 \cdot 10^5$ kg/cm^2 and $\nu_D = 0.35$) and epoxy ($E_e = 3.05 \cdot 10^4$ kg/cm^2 and $\nu_e = 0.41$) constituents; the specimens were fabricated according to the scheme shown in Fig. 2. The experiments were performed for 17 pairs of α and β values under the condition $\alpha + \beta = \pi$. The constituents were bonded by cyanoacrylate adhesive and epoxy resin at room temperature. The thickness of the joint layer δ was 0.03–0.07 mm and 0.05–0.10 mm for cyanoacrylate adhesive and epoxy resin, respectively. Six specimens were tested for each variant of joint angle and each type of glue. The total number of the specimens was 204. The error of technical treatment of the angles was $\pm 1°$.

The experimental setup designed and constructed by analogy with known setups [4] allows one to bring the specimens to failure by quasistatic loading, obtain the isochromatic stress patterns, and determine the rupture moments with an accuracy of 5–7%.

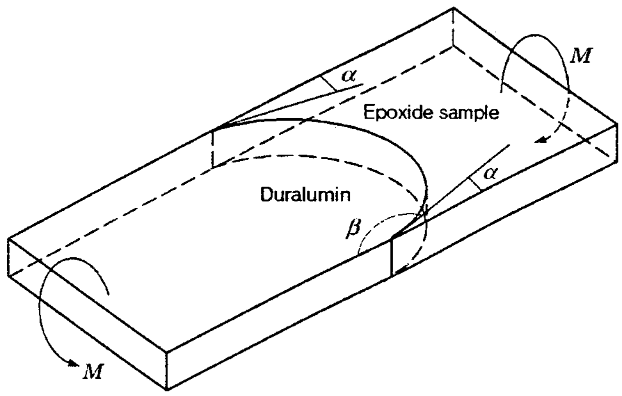

Fig. 2

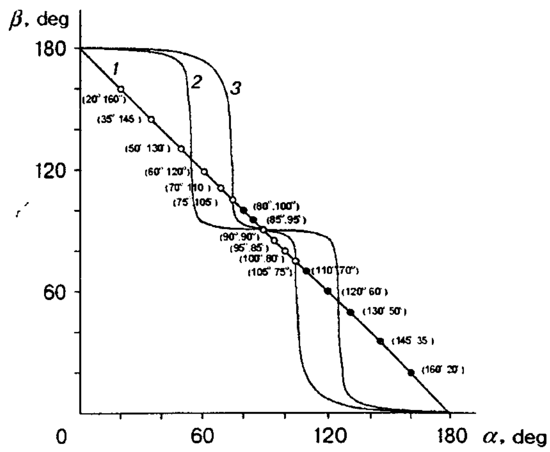

Fig. 3

In the experiments, the specimens were subjected to pure bending up to failure by continuous quasistatic loading. The values of the rupture moments (loads) were recorded for different specimens, and the isochromatic curves were photographed. After failure, the specimens were inspected to find the points at which the failure began and determine the kind of failure (adhesive or cohesive).

4. Discussion of Results. Figure 3 shows the limiting curves in the plane (α, β) that separate the low-stress and stress-concentration zones at the angular point of the specimen. Curves 1 and 2 are plotted with the use of Eq. (4) for $\gamma = 1$ and $\gamma = 23.28$, respectively, and curve *3* is plotted according to the experimental data ($\gamma = 23.28$) for the composite specimen bonded by the epoxy glue. There are points on curve 1 which correspond to combinations of the angles of the constituents at the angular points of the joints. The open points refer to the pairs of angles (α, β) at which low or finite stresses occur, and the filled points to the pairs of angles at which the stress concentration is observed at the angular points of the joints. The quantitative discrepancy between curves 2 and 3 is accounted for by the assumptions of incompressibility and the linear behavior of materials which were adopted in deriving Eq. (4).

Figure 4 shows the rupture moment M^*/M_0^* versus the angle α. Curves 1 and 2 refer to epoxy resin and cyanoacrylate adhesive, respectively. The dashed segments on curve 1 refer to low stresses at the angular point (M_0^* is the average value of the rupture moment for low stresses at the angular point). Curves 1 and 2 are shifted relative to each other by $\pm(5\text{–}15)°$.

The ultimate stress of the composite specimen is lowered by approximately a factor of 1.5 compared to the case of low stresses for angles of 75–90° (the first α interval, which corresponds to the stress concentrations

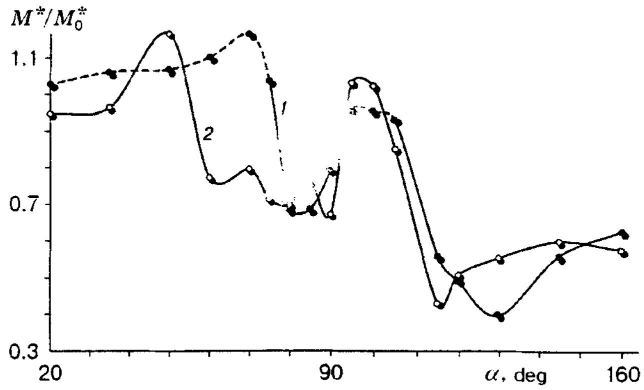

Fig. 4

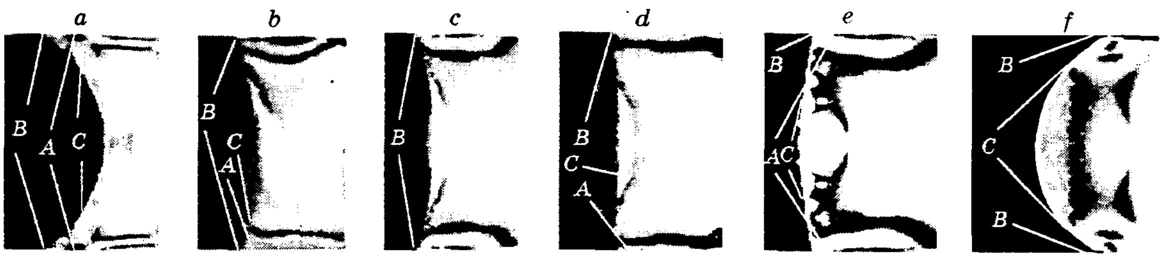

Fig. 5

at the angular points) and is lowered by approximately a factor of 2.4 for angles of 105–160° (the second interval of stress-concentration states).

Figure 5 shows typical isochromatic stress patterns (principal-stress curves) in the composite specimen for various α ($\alpha + \beta = \pi$). Figure 5a refers to the angles $\alpha = 20$, 35, 50, 60, and 70°, Fig. 5b to $\alpha = 75°$, Fig. 5c to $\alpha = 80$ and 85°, Fig. 5d to $\alpha = 90°$, Fig. 5e to $\alpha = 95$, 100, and 105°, and Fig. 5f to $\alpha = 110$, 120, 130, 145, and 160°.

The location of the point where the failure begins and its character depend on the ratio between α and β.

If the stress concentration occurs at the angular point of the joint (point B in Fig. 5c and f), the failure begins at this point and propagates toward the opposite angular point or point C, then toward the opposite point C through the epoxide, and further along the joint. In this case, the failure can be adhesive or adhesive-cohesive.

When the low stresses occur at the angular point, two types of failure are observed. In the first case, the failure propagates from point A on the lateral surface of the specimen (Fig. 5a) toward point C on the joint surface, then toward point C along the joint, and further through the epoxide toward the point symmetric with respect to point A. In the second case, the failure propagates from the internal point C on the joint surface (Fig. 5e) through the epoxide toward point A on the lateral surface and through the joint toward the point symmetric with respect to point C. Small pieces of cohesively failed epoxide remain at the angular points.

In the case where the stress concentration occurs at the angular points, the closed zero isochromatic curves (black curves) of different orders are concentrated in the neighborhood of the angular point; in the case of low stresses, the isochromatic curves are not closed and extend from the angular or the internal point throughout the specimen. Similar isochromatic stress patterns are given in [5, 6].

For the angles α that correspond to the points located near the limiting curve 3 (see Fig. 3), the stress concentration can occur at one angular point, whereas low stresses can occur at another symmetric point

(Fig. 5b). This can be attributed to the error of technical treatment of the angles at the angular points, which was $\pm 1^\circ$.

It should be noted that in the case where the constituents (Duralumin and epoxide) are bonded by epoxy resin, the experimental data allow one to distinguish clearly between the stress-concentration zones at the angular points and the low-stress zones in the (α, β) plane. In the case of cyanoacrylate adhesive, this can be done for α smaller than 50° (the low-stress region at the angular points) and in the interval of 120–160° (stress-concentration state). For other values of α, it is difficult to determine a distinct boundary between these states from the experimental data. Apparently, this is attributed to the fact that despite the small thickness of the cyanoacrylate layer, the specimen should be considered a three-component body (Duralumin, cyanoacrylate, and epoxide).

The authors are grateful to R. G. Melik-Yolchyan for his assistance in processing the patterns of the isochromatic curves.

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